# Distanţe

- *Distanţa dintre două puncte*

Distanţa dintre două puncte este segmentul de dreaptă ce uneşte cele două puncte.

- *Distanţa de la un punct la o dreaptă*

Distanţa de la un punct la o dreapta este lungimea perpendicularei duse din acest punct pe dreapta dată.

- *Distanţa de la un punct la un plan*

Prin distanţa de la un punct M la un plan α, inţelegem lungimea MN, unde N∈α este piciorul perpendicularei duse din M pe α.

- *Distanţa dintre două drepte paralele*

Distanţa dintre două drepte paralele este distanţa de la un punct de pe una din drepte la cealaltă dreptă.

- *Distanţa dintre două plane paralele*

Distanţa dintre două plane paralele este distanţa de la un punct dintr-un plan la celălalt plan.

- ✓ **Observaţie: Pentru calcularea distanţei de la un punct la o dreaptă construim perpendiculara din acel punct pe acea dreptă şi căutăm un triunghi eventual dreptunghic în care această distanţă să fie o latură sau linie importantă.**
- ✓ Observatie(2): Segmentul cel mai scurt de la un punct exterior unui plan la acel plan este segmentul perpendicular pe planul dat.

# Aplicaţii

**1)**

Ip. | ΔABC isoscel
AB=AC=15cm, BC=18cm
AM⊥(ABC), AM=$12\sqrt{3}$

C. | dist.(M, BC)=?
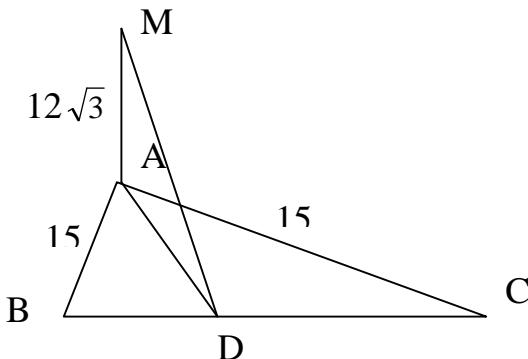
$12\sqrt{3}$ A 15 15 M B D C

Dem.:

Ducem AD⊥BC, D∈BC

$$
\left.
\begin{array}{l}
AM\perp(ABC)\\
AD\perp BC\\
AD\subset(ABC)\\
BC\subset(ABC)
\end{array}
\right\}
\xrightarrow{T.3.\perp} MD\perp BC \Rightarrow dist.(M,BC)=MD
$$

$$
\left.
\begin{array}{l}
\Delta ABC \text{ isoscel}\\
AD \text{ înălţime}
\end{array}
\right\}
\Rightarrow AD \text{ mediană} \Rightarrow
\left.
\begin{array}{l}
BD\equiv DC\\
\text{dar } BC=18
\end{array}
\right\}
\Rightarrow BD=DC=9
$$

AD⊥BC ⇒ ΔABD dreptunghic

$\Rightarrow AD^2=AB^2-BD^2$

$AD^2=225-81$

$AD^2=144$

$AD=12$

$$
\left.
\begin{array}{l}
AM\perp(ABC)\\
AD\subset(ABC)
\end{array}
\right\}
\Rightarrow AM\perp AD \Rightarrow \Delta MAD \text{ dreptunghic}
$$

$\Rightarrow MD^2=MA^2+AD^2$

$MD^2=144\cdot3+144$

$MD^2=144\cdot4$

$MD=24$

**2)**

| Ip. | $\triangle ABC$ dreptunghic( m(<A)=90°)<br>AM⊥(ABC), AM=3cm<br>AB=6cm, AC=$6\sqrt{3}$ |
|---|---|
| C. | dist.(M, BC)=? |



Dem.:
Ducem AD⊥BC, D∈BC
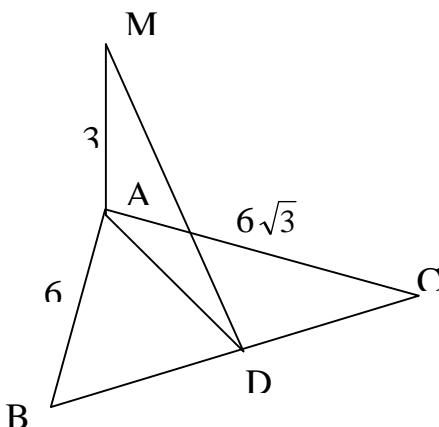
$\left.\begin{array}{l} AM\perp(ABC) \\ AD\perp BC \\ AD\subset(ABC) \\ BC\subset(ABC) \end{array}\right\}$ *T.3.⊥.* $\Rightarrow MD\perp BC \Rightarrow$ dist.(M,BC)=MD

$\left.\begin{array}{l} AM\perp(ABC) \\ AD\subset(ABC) \end{array}\right\} \Rightarrow AM\perp AD \Rightarrow \triangle MAD$ dreptunghic

$\triangle ABC$ dreptunghic
$\Rightarrow BC^2=AB^2+AC^2$
$\quad BC^2=36+108$
$\quad BC^2=144$
$\quad BC=12$

$\left.\begin{array}{l} AD\perp BC \Rightarrow AD \text{ înălţime} \\ \\ \triangle ABC \text{ dreptunghic} \end{array}\right\} \Rightarrow AD=\dfrac{AB\cdot AC}{BC} \Rightarrow AD=\dfrac{6\cdot 6\sqrt{3}}{12}$

$\Rightarrow AD=3\sqrt{3}$
$\triangle MAD$ dreptunghic
$\Rightarrow MD^2=AM^2+AD^2$
$\quad MD^2=9+27$
$\quad MD^2=25$
$\quad MD=5$

**3)**

Ip. | ABCD dreptunghi, AB=16cm, Bc=9cm
    | AM⊥(ABC), AM=12cm

C. | dist.(M, AB)=?
   | dist.(M, BC)=?
   | dist.(M, CD)=?
   | dist.(M, AD)=?
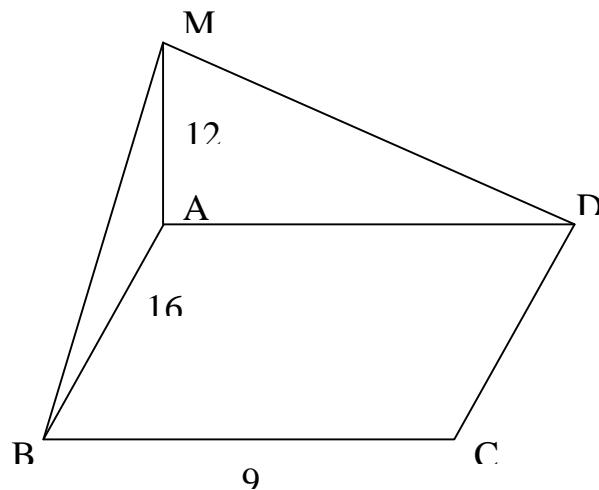


Dem.:

$\left.\begin{array}{l} AM\perp(ABC) \\ AD\subset(ABC) \end{array}\right\} \Rightarrow MA\perp AD \Rightarrow$ dist.(M, AD)=AM=12

$\left.\begin{array}{l} AM\perp(ABC) \\ AB\subset(ABC) \end{array}\right\} \Rightarrow MA\perp AB \Rightarrow$ dist.(M, AB)=AM=12

$\left.\begin{array}{l} AM\perp(ABC) \\ AD\perp DC \\ AD\subset(ABC) \\ DC\subset(ABC) \end{array}\right\} \overset{T.3.\perp.}{\Rightarrow} MD\perp DC \Rightarrow$ dist.(M, DC)=MD

$\left.\begin{array}{l} AM\perp(ABC) \\ AB\perp BC \\ AB\subset(ABC) \\ BC\subset(ABC) \end{array}\right\} \overset{T.3.\perp.}{\Rightarrow} MB\perp BC \Rightarrow$ dist.(M, BC)=MB

$MA\perp AD \Rightarrow \Delta MAD$ dreptunghic $\Rightarrow$ $MD^2=AM^2+AD^2$
$MD^2=144+81$
$MD^2=225$
$MD=15$

$MA\perp AB \Rightarrow \Delta MAB$ dreptunghic $\Rightarrow$ $MB^2=AM^2+AB^2$
$MB^2=144+256$
$MB^2=400$
$MB=20$

**4)**

Ip. | ABCD dreptunghi(AC∩BD={O}), AB=32cm, BC=18cm
OM⊥(ABC), OM=12cm

C. | dist.(M, AB)=?
dist.(M, BC)=?
dist.(M, CD)=?
dist.(M, AD)=?
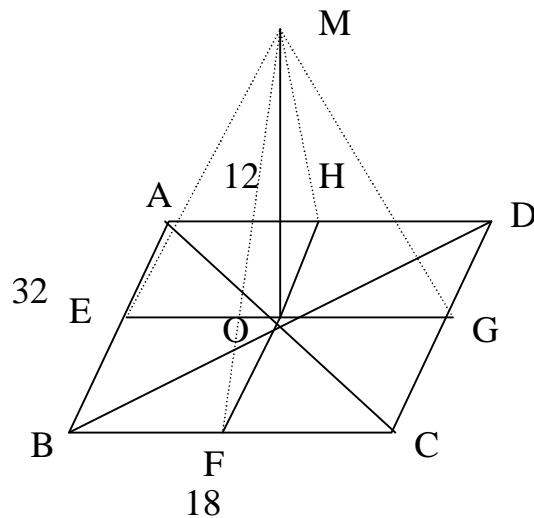


Dem.:

Ducem OE⊥AB, E∈AB

OF⊥BC, F∈BC

OG⊥DC, G∈DC

OH⊥AD, H∈AD

OM⊥(ABC)  
OE⊥AB  *T.3.⊥*  
OE⊂(ABC) $\Rightarrow$ ME⊥AB $\Rightarrow$ dist.(M, AB)=ME  
AB⊂(ABC)

OM⊥(ABC)  
OF⊥BC  *T.3.⊥*  
OF⊂(ABC) $\Rightarrow$ MF⊥BC $\Rightarrow$ dist.(M, BC)=MF  
BC⊂(ABC)

OM⊥(ABC)  
OG⊥CD  *T.3.⊥*  
OG⊂(ABC) $\Rightarrow$ MG⊥AB $\Rightarrow$ dist.(M, CD)=MG  
CD⊂(ABC)

OM⊥(ABC)  
OH⊥AD  *T.3.⊥*  
OH⊂(ABC) $\Rightarrow$ MH⊥AD $\Rightarrow$ dist.(M, AD)=MH  
AD⊂(ABC)

ABCD dreptunghi $\Rightarrow$ AO≡OC  
BO≡OD  $\Rightarrow$ ΔAOB, ΔBOC, ΔCOD, ΔAOD isoscele  
AC≡BD

$\Delta AOB$ isoscel $\Big\}\Rightarrow$ OE mediană $\Rightarrow$ AE≡EB $\Big\}\Rightarrow$ AE=EB=16
OE inalţime $\quad\Big\}\qquad\qquad\qquad\qquad$ AB=32

$\Delta BOC$ isoscel $\Big\}\Rightarrow$ OF mediană $\Rightarrow$ BF≡FC $\Big\}\Rightarrow$ BF=FC=9
OF inalţime $\quad\Big\}\qquad\qquad\qquad\qquad$ BC=18

$\Delta COD$ isoscel $\Big\}\Rightarrow$ OG mediană $\Rightarrow$ CG≡GD $\Big\}\Rightarrow$ CG=GD=16
OG inalţime $\quad\Big\}\qquad\qquad\qquad\qquad$ CD=32

$\Delta AOD$ isoscel $\Big\}\Rightarrow$ OH mediană $\Rightarrow$ DH≡HA $\Big\}\Rightarrow$ AH=HA=9
OH inalţime $\quad\Big\}\qquad\qquad\qquad\qquad$ AD=18

OE⊥AB $\Big\}\Rightarrow$ AD∥EO
AD⊥AB $\quad\Big\}\qquad\qquad\Bigg\}\Rightarrow$ AEON paralelogram $\Rightarrow$ OE=9
OE⊥AE $\Big\}\Rightarrow$ AE∥ON
OE⊥ON $\quad\Big\}$

OF⊥BC $\Big\}\Rightarrow$ AB∥OF
AB⊥BC $\quad\Big\}\qquad\qquad\Bigg\}\Rightarrow$ EBFO paralelogram $\Rightarrow$ OF=16
OE⊥AB $\Big\}\Rightarrow$ OE∥BF
FB⊥AB $\quad\Big\}$

OG⊥DC $\Big\}\Rightarrow$ OG∥FC
FC⊥DC $\quad\Big\}\qquad\qquad\Bigg\}\Rightarrow$ OFCG paralelogram $\Rightarrow$ OG=9
OF⊥BC $\Big\}\Rightarrow$ GC∥OG
GC⊥BC $\quad\Big\}$

ON⊥AD $\Big\}\Rightarrow$ ON∥GD
CD⊥AD $\quad\Big\}\qquad\qquad\Bigg\}\Rightarrow$ NOGD paralelogram $\Rightarrow$ OE=16
ND⊥DC $\Big\}\Rightarrow$ ND∥OG
OG⊥DG $\quad\Big\}$

$\Delta MOE$ dreptunghic $\Rightarrow$ $ME^2=OM^2+OE^2$
$ME^2=144+81$
$ME^2=225 \Rightarrow ME=15$

$\Delta MOF$ dreptunghic $\Rightarrow$ $MF^2=OM^2+OF^2$
$MF^2=144+256$
$MF^2=400 \Rightarrow MF=20$

$\Delta MOG$ dreptunghic $\Rightarrow$ $MG^2=OM^2+OG^2$
$MG^2=144+81$
$MG^2=225 \Rightarrow MG=15$

$\Delta MOH$ dreptunghic $\Rightarrow$ $MH^2=OM^2+OH^2$
$MH^2=144+256$
$MH^2=400 \Rightarrow MG=20$