DETERMINANTI TRIGONOMETRICI

A) Unele proprietati si reguli de calculare a determinantilor:

1) $\det A = \det A^T$

2)Daca intr-un determinant de ordinul n, elementele de pe o linie, coloana sunt 0 atunci valoarea determinantului este 0.

3)Daca intr-un determinant 2 linii,coloane sunt proportionale atunci valoarea determinantului este 0.

4)Daca intr-un determinant schimbam 2 linii,coloane atunci determinantul nou obtinut este =”- ” determinantul initial.

5)Complementul algebric:

$$\delta_{ij} = (-1)^{i+j} d_{ij}$$

Regula lui Laplace pentru dezvoltarea determinantului de ordinul n dupa o linie,coloana:

$$d_n = a_{i1}\delta_{i1} + a_{i2}\delta_{i2} + ... + a_{in}\delta_{in}$$

$$d_n = a_{1j}\delta_{1j} + a_{2j}\delta_{2j} + ... + a_{nj}\delta_{nj}$$

6)Determinant Vandermonde:

$$V(a_1,...,a_n) = \begin{vmatrix} 1 & 1 & .... & 1 \\ a_1 & a_2 & .... & a_n \\ a_1^2 & a_2^2 & .... & a_n^2 \\ .... & .... & .... & .... \\ a_1^{n-1} & a_2^{n-2} & .... & a_n^{n-1} \end{vmatrix}$$

:

$$V(a_1,...,a_n) = \prod_{1 \le j < i \le n} (a_i - a_j)$$

B)Formule trigonometrice folosite:

$$1)\ \sin^2\alpha+\cos^2\alpha=1$$
$$2)\ \sin(\alpha+-\beta)=\sin\alpha\cos\beta+-\sin\beta\cos\alpha$$
$$3)\ \cos(\alpha+-\beta)=\cos\alpha\cos\beta-+\sin\alpha\sin\beta$$

$$4)\ \sin 2\alpha=2\sin\alpha\cos\alpha$$
$$5)\ \cos 2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1$$
$$6)\ \sin 3\alpha=3\sin\alpha-4\sin^3\alpha$$
$$7)\ \cos 3\alpha=4\cos^3\alpha-3\cos\alpha$$
$$8)\ \sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$9)\ \sin\alpha-\sin\beta=2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$$
$$10)\ \cos\alpha+\cos\beta=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$
$$11)\ \cos\alpha-\cos\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\beta-\alpha}{2}$$
$$12)\ \sin\frac{A}{2}=\sqrt{\frac{(p-b)(p-c)}{bc}}$$
$$13)\ \cos\frac{A}{2}=\sqrt{\frac{p(p-a)}{bc}}$$
$$14)\ \cos ecA=\frac{2R}{a}$$

APLICATII

Sa se calculeze determinantii:

$$1)\ \Delta=\begin{vmatrix}\sin^2 a & \cos^2 a & \sin a\cos a\\ \sin^2 b & \cos^2 b & \sin b\cos b\\ \sin^2 c & \cos^2 c & \sin c\cos c\end{vmatrix}$$

$$\Delta=\cos^2 a\cos^2 b\cos^2 c\begin{vmatrix}tg^2a & 1 & tga\\ tg^2b & 1 & tgb\\ tg^2c & 1 & tgc\end{vmatrix}$$

$$\Delta=\cos^2 a\cos^2 b\cos^2 c\begin{vmatrix}1 & tga & tg^2a\\ 1 & tgb & tg^2b\\ 1 & tgc & tg^2c\end{vmatrix}$$

$$\overset{VANDER}{\Rightarrow}\quad \Delta=\cos^2 a\cos^2 b\cos^2 c(tgb-tga)(tgc-tga)(tgc-tgb)$$

$$2)\Delta=\begin{vmatrix}1 & 1 & 1\\ \cos a & \cos b & \cos c\\ \cos 2a & \cos 2b & \cos 2c\end{vmatrix}$$

$c_1(-1)+c_2$
$c_1(-1)+c_3$

$$\Delta=\begin{vmatrix}1 & 0 & 0\\ \cos a & \cos b-\cos a & \cos c-\cos a\\ \cos 2a & \cos 2b-\cos 2a & \cos 2c-\cos 2a\end{vmatrix}$$

$$\Delta=\begin{vmatrix}\cos b-\cos a & \cos c-\cos a\\ 2\cos^2 b-1-2\cos^2 a+1 & 2\cos^2 c-1-2\cos^2 a+1\end{vmatrix}$$

$$\Delta=2\begin{vmatrix}\cos b-\cos a & \cos c-\cos a\\ (\cos b-\cos a)(\cos b+\cos a) & (\cos c-\cos a)(\cos c+\cos a)\end{vmatrix}$$

$$\Delta=2(\cos b-\cos a)(\cos c-\cos a)\begin{vmatrix}1 & 1\\ \cos b+\cos a & \cos c+\cos a\end{vmatrix}$$

$$\Delta=2(\cos b-\cos a)(\cos c-\cos a)(\cos c-\cos b)$$

$$3)\Delta=\begin{vmatrix}1 & \cos a & \cos 2a\\ \cos a & \cos 2a & \cos 3a\\ \cos 2a & \cos 3a & \cos 4a\end{vmatrix}$$

l_1+l_3

$$\Delta=\begin{vmatrix}1 & \cos a & \cos 2a\\ \cos a & \cos 2a & \cos 3a\\ \cos 2a+1 & \cos 3a+\cos a & \cos 4a+\cos 2a\end{vmatrix}$$

$$\Delta=\begin{vmatrix}1 & \cos a & \cos 2a\\ \cos a & \cos 2a & \cos 3a\\ 2\cos^2 a & 2\cos 2a\cos a & 2\cos 3a\cos a\end{vmatrix}$$

$Liniile$ 2 si 3 sunt proportionale $\Rightarrow \Delta=0$

$$4)\Delta = \begin{vmatrix} 1 & tg\frac{B}{2}tg\frac{C}{2} & \cos ecB\cos ecC \\ 1 & tg\frac{C}{2}tg\frac{A}{2} & \cos ecC\cos ecA \\ 1 & tg\frac{A}{2}tg\frac{B}{2} & \cos ecA\cos ecB \end{vmatrix}$$

$$tg\frac{B}{2}tg\frac{C}{2} = \frac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}} = \frac{\sqrt{\frac{(p-a)(p-c)}{ac}}\sqrt{\frac{(p-a)(p-b)}{ab}}}{\sqrt{\frac{p(p-b)}{ac}}\sqrt{\frac{p(p-c)}{ab}}} = \sqrt{\frac{(p-a)^2}{p^2}} = \frac{p-a}{p}$$

$$\Rightarrow tg\frac{C}{2}tg\frac{A}{2} = \frac{p-b}{p}$$

$$tg\frac{A}{2}tg\frac{B}{2} = \frac{p-c}{p}$$

$$\mathrm{cosecA} = \frac{\mathrm{2R}}{\mathrm{a}}$$

$$\Delta = \begin{vmatrix} 1 & \frac{p-a}{p} & \frac{4R^2}{bc} \\ 1 & \frac{p-b}{p} & \frac{4R^2}{ac} \\ 1 & \frac{p-c}{p} & \frac{4R^2}{ab} \end{vmatrix}$$

$l_1(-1)+l_2$

$l_1(-1)+l_3$

$$\Delta = \begin{vmatrix} 1 & \frac{p-a}{p} & \frac{4R^2}{bc} \\ 0 & \frac{a-b}{p} & \frac{4R^2(b-a)}{abc} \\ 0 & \frac{a-c}{p} & \frac{4R^2(c-a)}{abc} \end{vmatrix} = \begin{vmatrix} \frac{a-b}{p} & \frac{4R^2(b-a)}{abc} \\ \frac{a-c}{p} & \frac{4R^2(c-a)}{abc} \end{vmatrix}$$

$$\Delta = \frac{4R^2}{abcp}\begin{vmatrix} a-b & b-a \\ a-c & c-a \end{vmatrix} = 0$$

$$5)\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & \cos c & \cos b \\ 1 & \cos c & 1 & \cos a \\ 1 & \cos b & \cos a & 1 \end{vmatrix}$$

$c_1(-1) + c_2$
$c_1(-1) + c_3$
$c_1(-1) + c_4$

$$\Delta = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & \cos c - 1 & \cos b - 1 \\ 1 & \cos c - 1 & 0 & \cos a - 1 \\ 1 & \cos b - 1 & \cos a - 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & \cos c - 1 & \cos b - 1 \\ \cos c - 1 & 0 & \cos a - 1 \\ \cos b - 1 & \cos a - 1 & 0 \end{vmatrix}$$

$$\cos x - 1 = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} - 1 = 1 - 2\sin^2 \frac{x}{2} - 1 = -2\sin^2 \frac{x}{2}$$

$$\Delta = \begin{vmatrix} 0 & -2\sin^2 \frac{c}{2} & -2\sin^2 \frac{b}{2} \\ -2\sin^2 \frac{c}{2} & 0 & -2\sin^2 \frac{a}{2} \\ -2\sin^2 \frac{b}{2} & -2\sin^2 \frac{a}{2} & 0 \end{vmatrix}$$

$$\Delta = -8\sin \frac{a}{2} \sin \frac{b}{2} \sin \frac{c}{2} - 8\sin \frac{a}{2} \sin \frac{b}{2} \sin \frac{c}{2} = -16\sin \frac{a}{2} \sin \frac{b}{2} \sin \frac{c}{2}$$

6) *Sa* se arate ca :

$$\begin{vmatrix} 1 & x & x^3 \\ \cos n\theta & \cos(n+1)\theta & \cos(n+3)\theta \\ \sin n\theta & \sin(n+1)\theta & \sin(n+3)\theta \end{vmatrix}$$ este divizibil cu $1 - 2x\cos\theta + x^3$

Dezvoltam dupa prima linie :

$$\Delta = (-1)^{1+1} \begin{vmatrix} \cos(n+1)\theta & \cos(n+3)\theta \\ \sin(n+1)\theta & \sin(n+3)\theta \end{vmatrix} + (-1)^{1+2} \begin{vmatrix} \cos n\theta & \cos(n+3)\theta \\ \sin n\theta & \sin(n+3)\theta \end{vmatrix} + (-1)^{1+3} \begin{vmatrix} \cos n\theta & \cos(n+1)\theta \\ \sin n\theta & \sin(n+1)\theta \end{vmatrix}$$

$$\Delta = \cos(n+1)\theta \sin(n+3)\theta - \cos(n+3)\theta \sin(n+1)\theta - x[\cos n\theta \sin(n+3)\theta - \cos(n+3)\theta \sin n\theta] + \\ + x^3[\cos n\theta \sin(n+1)\theta - \cos(n+1)\theta \sin n\theta]$$

$$\Delta = \sin[(n+3)\theta - (n+1)\theta] - x\sin[(n+3)\theta - n\theta] + x^3 \sin[(n+3)\theta - n\theta]$$

$$\Delta = \sin 2\theta - x\sin 3\theta + x^3 \sin\theta$$

$$\Delta = 2\sin\theta\cos\theta - x[3\sin\theta - 4\sin^3\theta] + x^3\sin\theta$$

$$\Delta = \sin\theta[2\cos\theta - x(3 - 4\sin^2\theta) + 2\cos\theta]$$

$$\Delta = \sin\theta[x^3 - x(3 - 4 + 4\cos^2\theta) + 2\cos\theta]$$

$$\Delta = \sin\theta[x^3 + x(1 - 4\cos^2\theta) + 2\cos\theta]$$

$$\Delta = \sin\theta[x^3 - 4x\cos^2\theta + x + 2\cos\theta]$$

$$\Delta = \sin\theta[x(x^2 - 4\cos^2\theta) + x + 2\cos\theta]$$

$$\Delta = \sin\theta[x(x - 2\cos\theta)(x + 2\cos\theta) + x + 2\cos\theta]$$

$$\Delta = \sin\theta(x + 2\cos\theta)[x^2 - 2x\cos\theta + 1] \Rightarrow x^2 - 2x\cos\theta + 1 \,|\, \Delta$$

7)Sa se arate ca, daca a, b, c, sunt unghiuri ascutite, atunci :

$$\begin{vmatrix} \sqrt{1+\cos a} & \sqrt{1+\sin a} & \sqrt{1-\sin a} \\ \sqrt{1+\cos b} & \sqrt{1+\sin b} & \sqrt{1-\sin b} \\ \sqrt{1+\cos c} & \sqrt{1+\sin c} & \sqrt{1-\sin c} \end{vmatrix} = 0$$

$$1+\cos x = 1+\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1 + 2\cos^2\frac{x}{2} - 1 = 2\cos^2\frac{x}{2}$$

$$1+\sin x = \cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2} = (\sin\frac{x}{2} + \cos\frac{x}{2})^2$$

$$1-\sin x = (\sin\frac{x}{2} - \cos\frac{x}{2})^2$$

8)Sa se arate ca :

$$D=\begin{vmatrix} 1+\alpha\cos\phi & \alpha^2\cos2\phi & \alpha^3\cos3\phi & .... & \alpha^n\cos n\phi \\ \alpha\cos\phi & 1+\alpha^2\cos2\phi & \alpha^3\cos3\phi & .... & \alpha^n\cos n\phi \\ \alpha\cos\phi & \alpha^2\cos2\phi & 1+\alpha^3\cos3\phi & .... & \alpha^n\cos n\phi \\ .... & .... & .... & .... & .... \\ \alpha\cos\phi & \alpha^2\cos2\phi & \alpha^3\cos3\phi & .... & 1+\alpha^n\cos n\phi \end{vmatrix}=$$

$$=\frac{\alpha^{n+2}\cos n\phi-\alpha^{n+1}\cos(n+1)\phi+1}{\alpha^2-2\alpha\cos\phi+1}$$

Notam $a_k=\alpha^k\cos k\phi \quad k=1,2,...,n$

$$D=\begin{vmatrix} 1+a_1 & a_2 & a_3 & .... & a_n \\ a_1 & 1+a_2 & a_3 & .... & a_n \\ a_1 & a_2 & 1+a_3 & .... & a_n \\ ... & .... & .... & .... & .... \\ a_1 & a_2 & a_3 & .... & 1+a_n \end{vmatrix}$$

c_1+c_2

c_1+c_3

....

c_1+c_n

$D=(1+a_1)$