Modelul Phillips

Aplicatie:

Avem urmatoarele date estimate :
-propensitatea spre economisire s =0,3;
-propensitatea marginala a consumului final si pentru investitii b = 0,7;
-coeficientul de ajustare a nivelului real al cheltuielilor guvernamentale G_t la nivelul dorit $\hat{G}(t)$ α =3 ;
-parametru de reactie al variatiei ofertei la variatia cererii excedentare k = 5

Cereri:

a) Cuantificati efectul unei descresteri unitare a cererii agregate(u = 1).
b) Cercetati consecintele politicii de stabilizare de tip:
   i) proportional cu γ_P = 0,2
   ii) diferentiala cu γ_D = 0,1
   iii) integrala cu γ_I = 0,25
   iv) proportional –diferentiala cu γ_P = 0,2 si γ_D = 0,1
   v) proportional-integrala cu γ_P = 0,2 si γ_I = 0,25
   vi) diferential –integrala cu γ_D = 0,1 si γ_I = 0,25

Avem urmatoarele date initiale: Y(0) = 2000, $\dot{Y}(0) = -400$, $\ddot{Y}(0)$ = -2000

Rezolvare:

a) Fie D(t) cererea agregata pe piata bunurilor si serviciilor , avem:

D(t) = bY(t) +G(t) - u (1)

c = kE(t) = k(D(t)-Y(t)), k>0 (2)

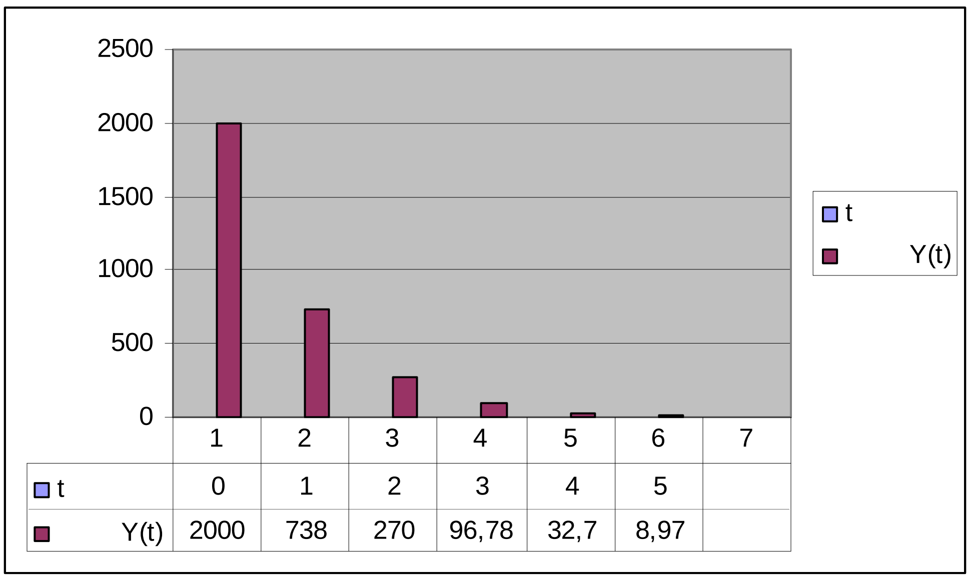
unde Y(t)-oferta agregata
E(t)- functia excesului de cerere

Din (1) si (2) $\Rightarrow \begin{cases} \dot{Y}(t) = 5E(t) \\ D(t) = 0,7Y(t) - u, \end{cases}$ cu u =1

$\Rightarrow \dot{Y}(t) = 5[0,7Y(t) - 1 - Y(t)] \Rightarrow \dot{Y}(t) = -Y(t) - 5$

Avem urmatoarea evolutie: $Y(t) = e^{-t}(Y_0 - Y^*) + Y^*$ unde $Y^* = -5 \Rightarrow Y(t) = 20005e^{-t}$ -5;
Astfel se observa ca descresterea cererii cu o unitate duce la o deplasare a echilibrului mai jos cu 5 unitati ale ofertei

t	Y(t)
0	2000
1	738
2	270
3	96,78
4	32,7
5	8,97



Avem:

$$\dot{D}(t) + \alpha D(t) = b\dot{Y}(t) + \dot{G}(t) + \alpha bY(t) + \alpha G(t) - \alpha u \qquad (3)$$

conditia de echilibru sub aspectul cererii

$$\dot{D}(t) + \alpha D(t) = \frac{1}{k}\left[\ddot{Y}(t) + k\dot{Y}(t) + \alpha\dot{Y}(t) + \alpha kY(t)\right] \qquad (4)$$

conditia de echilibru sub aspectul ofertei

Din (3) si (4) deducem ca:

$\ddot{Y}(t) = -7{,}7\dot{Y}(t) - 7$Y (t) +15 G$^{*}(t)$- 15 u , unde u =1.

$\lambda_1 = -\alpha = -3, \lambda_2 = -ks = -1{,}5$

Y(t) = Y$^{G}(t) + Y^{*}$= A$_1$e$^{\lambda_1 t}$$+A_2 e^{\lambda_2 t}$+ Y*

Coeficientii $A_{1,} A_2$ se determina din datele initiale:

t = 0$\Rightarrow$ $A_{1,} + A_2$ +`Y*=Y(0)

Derivand, obtinem : $\dot{Y}(t) =$ A$_1$ λ e$^{\lambda_1 t}$$+A_2 \lambda e^{\lambda_2 t}$

$$\begin{cases} A_1 + A_2 = Y(0) - Y^{*} \\ A_1\lambda_1 + A_2\lambda_2 = \dot{Y}(0) \end{cases}$$

unde Y$^{*} = -2{,}14$

$$\begin{cases} A_1 + A_2 = 2002{,}14 \\ -3A_1 - 1{,}5A_2 = -400 \end{cases}$$

Obtinem:

$$\begin{cases} A_1 = -1735{,}47 \\ A_2 = 3737{,}61 \end{cases}$$

Deci Y(t) = -1735,47 e $^{-3t}$+3737,61e$^{-1{,}5t}$ -2,14

b1) Avem:

$\hat{G}(t) = -\gamma_P Y(t) = -0{,}2Y(t)$

Deci ecuatia ofertei agregate devine :

$\ddot{Y}(t) = -7{,}7\dot{Y}(t) - 4$Y(t) – 15

Ecuatia caracteristica:

$\lambda^2 + 7{,}7\lambda + 4 = 0$

$\Delta = 7{,}7^2 - 4 * 4 = 59{,}3 - 16 = 43{,}3 > 0 \Rightarrow \lambda_{1,2} \in \Re$, cu $\lambda_1, \lambda_2 < 0$

$$\lambda_{1,2} = \frac{-7{,}7 \pm 6{,}6}{2} \quad \begin{cases} \lambda_1 = -0{,}55 \\ \lambda_2 = -7.15 \end{cases}$$

Y(t) = $Y^G(t) + Y^* = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + Y^*$

Coeficientii $A_{1,} A_2$ se determina din datele initiale:

t = 0 $\Rightarrow A_{1,} + A_2$ +\`Y^* =Y(0) (5)

Derivand, obtinem : $\dot{Y}(t) = A_1 \lambda_1 e^{\lambda_1 t} + A_2 \lambda_2 e^{\lambda_2 t}$ (6)

Din (5) si (6) obtinem sistemul

$$\begin{cases} A_1 + A_2 = Y(0) - Y^* \\ A_1\lambda_1 + A_2\lambda_2 = \dot{Y}(0) \end{cases}$$

unde $Y^* = -3{,}75$

prin inlocuire obtinem

$$\begin{cases} A_1 + A_2 = 2003{,}75 \\ -0{,}55A_1 - 7{,}15A_2 = -400 \end{cases}$$

Obtinem:

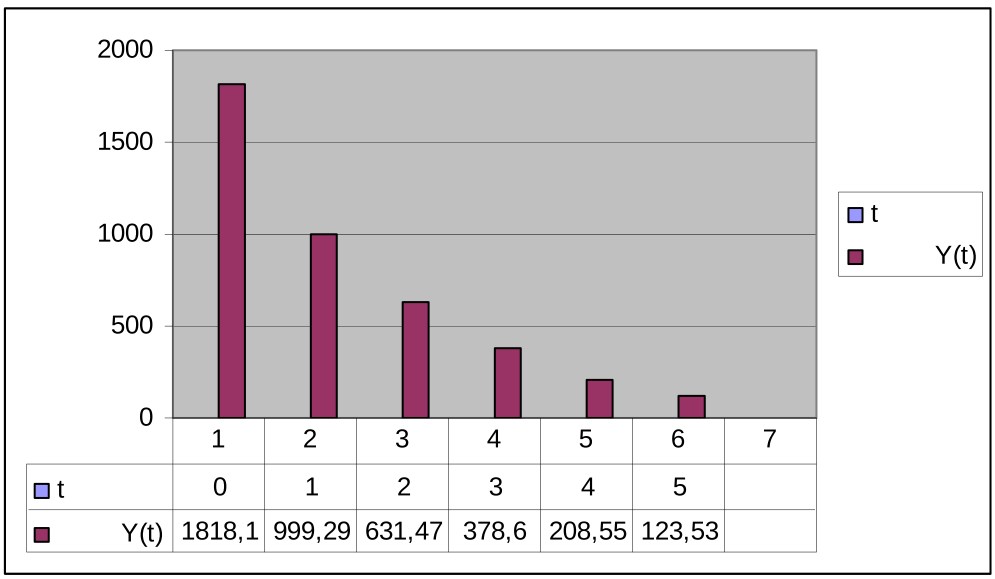
$$\begin{cases} A_1 = 1913{,}018 \\ A_2 = -91{,}21 \end{cases}$$

Deci Y(t) = $1913{,}018\, e^{-0{,}55t} - 91{,}21\, e^{-7{,}15t}$ -3,75

Deci traiectoria de evolutie este stabila(monoton convergenta) pentru ca e $^{\lambda_i t} \to 0$, i=1,2 $\Rightarrow$ Y(t) $\to Y^*$.

Se constata o reducere a ofertei numai cu -3,75 unitati datorita reducerii cererii cu o unitate , fata de cazul initial cand nu se asigura o politica guvernamentala de stabilizare, cand reducerea era de 5 unitati.

t	Y(t)
0	1818,058
1	999,29
2	631,47
3	378,6
4	208,55
5	123,53



Deoarece $a = -\frac{\alpha + ks}{2} = -2{,}25 < 0$, traiectoria de evolutie a lui Y(t) va fi stabila (oscilant amortizata cu perioada T=$\frac{2\pi}{b} = 8{,}97$ ani).

In concluzie, efectul unui control proportional prin intermediul variabilei cheltuielilor guvernamentale dorite $\hat{G}(t)$ este obtinerea, in toate cazurile, a unei traiectorii de evolutie stabile Y(t), catre sau in jurul traiectoriei de echilibru stationar $\bar{Y}(t)$

$\bar{Y}(t)$ se determina, in toate cazurile in functie de ipoteza care se face asupra naturii socurilor si perturbatiilor, data de forma variabilei u.

Astfel, daca presupunem ca u = 1 deci cererea agregata ar descreste cu 1 u.m, atunci

$\bar{Y}(t) = \bar{Y} = ct$

Din relatia $\ddot{Y}(t) + (\alpha + ks)\dot{Y}(t) + \alpha k(s + \gamma_P)Y(t) = -\alpha ku$ avem:

$\alpha k(s + \gamma_P)\bar{Y} = -\alpha k$

Deci :

$$\bar{Y} = -\frac{\alpha k}{\alpha k(s + \gamma_P)} = -\frac{1}{s + \lambda_P}$$

Deoarece $\left|-\frac{1}{s + \lambda_P}\right| < \left|-\frac{1}{s}\right| = \left|-\frac{1}{0,3}\right| = 3,33$, rezulta ca Y(t) va descreste mai incet sub actiunea controlului proportional decat in absenta acestuia.

b $_2$) In acest caz $\hat{G}(t)$ este inlocuit, in expresia :

$\ddot{Y}(t) + (\alpha + ks)\dot{Y}(t) + \alpha ksY(t) = \alpha k\hat{G}(t) - \alpha ku$, de relatia

$\hat{G}(t)$ = - $\lambda_P\,\dot{Y}(t)$ deci obtinem:

$\ddot{Y}(t) + [\alpha + ks - \alpha k\gamma_D]\dot{Y}(t) + \alpha ksY(t) = -\alpha ku$

Ecuatia caracteristica asociata ecuatiei diferentiale omogene ce se obtine din (7) este:

$\lambda^2 + [\alpha + ks - \alpha k\gamma_D]\lambda + \alpha ks = 0 \Rightarrow \lambda^2 + 3\lambda + 4,5 = 0 \Rightarrow \Delta = 9 - 18$= -9<0, deci $\lambda_{1,2} \in C$

Avand radacinile

$\lambda_{1,2} = -\frac{3}{2} \pm \frac{3i}{2}$, astfel incat evolutia ofertei aggregate este:

Y(t) = $Y^G(t) + Y^*$ = $A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + Y^*$,

In care

$e^{\lambda_1 t} = e^{-1,5t} e^{1,5it} = e^{-1,5t}\left(\cos\frac{3}{2}t + i\sin\frac{3}{2}t\right)$

$e^{\lambda_2 t} = \quad e^{-1,5t}\left(\cos\frac{3}{2}t - i\sin\frac{3}{2}t\right)$

si $Y^* = -\frac{4,5}{3} = -1,5$

Coeficientii $A_{1,} A_2$ se determina din datele initiale:

t = 0 $\Rightarrow A_{1,} + A_2$ +`Y^* =Y(0) (5)

Derivand, obtinem : $\dot{Y}(t) = A_1 \lambda_1 e^{\lambda_1 t} + A_2 \lambda_2 e^{\lambda_2 t}$ (6)

Din (5) si (6) obtinem sistemul

$$\begin{cases} A_1 + A_2 = Y(0) - Y^* \\ A_1\lambda_1 + A_2\lambda_2 = \dot{Y}(0) \end{cases}$$

prin inlocuire obtinem

$$\begin{cases} A_1 + A_2 = 2001,5 \\ (-1,5 + 1,5i)A_1 + (-1,5 - 1,5i)A_2 = -400 \end{cases}$$

Obtinem:

$$\begin{cases} A_1 = 1000 - 866i \\ A_2 = 1000 + 866i \end{cases}$$

Avem $\rho = |A_1| = \sqrt{1000^2 + 866^2} = 1322,87$

Y(t) = $Y^G(t) + Y^*$ = (1000-866i)$e^{(-1,5+1,5i)t}$ +$(1000 + 866i)$ $e^{(-1,5-1,5i)t}$ -1,5

Cum a = -1,5<0 $\Rightarrow$ Dinamica lui Y(t)este de tip stationar (oscilant amortizata), deci cand $\gamma_D = 0,1 < \frac{\alpha + ks}{ks} = 0,3$

Cum $\gamma_D \in \left(\frac{(\sqrt{\alpha}-\sqrt{ks})^2}{\alpha k}, \frac{\alpha+ks}{\alpha k} \right)$=(0,01;0,3) , dinamica lui Y(t) este oscilant amortizata in jurul traiectoriei de echilibru sationar Y^*.

b3)Cheltuielile guvernamentale dorite $\hat{G}(t)$ sunt date de relatia

$\hat{G}(t) = -\gamma_I \int_0^t Y(\tau)d\tau$ deci, ecuatia $\ddot{Y}(t)+(\alpha+ks)\dot{Y}(t)+\alpha ksY(t)=\alpha k\hat{G}(t)-\alpha ku$ devine

$$\ddot{Y}(t)+(\alpha+ks)\dot{Y}(t)+\alpha ksY(t)=-\alpha k\gamma_I \int_0^t \dot{Y}(\tau)d\tau-\alpha ku$$

Diferentiem cu timpul intreaga relatie, obtinem

$$\dddot{Y}(t)+(\alpha+ks)\ddot{Y}(t)+\alpha ks\dot{Y}(t)=-\alpha k\gamma_I(Y(t)-Y_0)$$

$$\dddot{Y}(t)+4{,}5\ddot{Y}(t)+4.5\dot{Y}(t)+3{,}75\ Y(t)-3{,}75Y_0=0$$

conditia de echilibru luand forma unei ecuatii diferentiale liniare si neomogene de ordin trei.

Ecuatia caracteristica asociata ecuatiei omogene obtinuta se scrie

$$\lambda^3+[\alpha+ks]\lambda^2+\alpha ks\ \lambda+\alpha k\gamma_I=0$$

$$\lambda^3+4{,}5\lambda^2+4{,}5\ \lambda+3{,}75\ =0$$

Pentru analiza stabilitatilor radacinilor ecuatiei se poat eutiliza criteriul matricial de stabilitate al lui Routh-Hurwitz.Conform acestuia, ecuatia algebrica de gradul trei

$$a_0\lambda^3+a_1\lambda^2+a_2\lambda+a_3=0$$

$$\lambda^3+4{,}5\lambda^2+4{,}5\ \lambda+3{,}75\ =0$$

are radacinile subunitare (positive sau negative), daca matricea de stabilitate

$$S=\begin{pmatrix} a_1a_3a_5 \\ a_0a_2a_4 \\ 0a_1a_3 \end{pmatrix}$$

are toti minorii principali pozitivi ($\Delta_i>0$) .

Astfel:

$$S=\begin{pmatrix} 4,5 & 3,75 & 0 \\ 1 & 4,5 & 0 \\ 0 & 4,5 & 3,75 \end{pmatrix}$$

$\Delta_1 = \alpha + ks = 4,5 > 0$

$$\Delta_2 = \begin{vmatrix} 4,5 & 3,75 \\ 1 & 4,5 \end{vmatrix} = 16,5>0$$

$\Delta_3 = 3,75 * 16,5 = 61,87 > 0$

$\lambda_1 = -3,3 \Rightarrow \lambda^2 + 1,2\lambda + 1,13 = 0$, cu $\lambda_{2,3} \approx -0,6 \pm 0,87i$

Evolutia ofertei agregate este data de :

$$Y(t) = A_1 e^{-3,3t} + 2\rho\, e^{-0,6t} \cos(0,87t+\varphi) + Y^*$$

unde $\rho = | c_2 |$, $\varphi = \arg c_2$

t=0 $\Rightarrow A_1 + 2\rho \cos\varphi = Y_0 - Y^*$

Dupa ce derivam, obtinem:

$\dot{Y}(t) = -3,3A_1 e^{-3,3t} - 1,2\rho\, e^{-0,6t} \cos(0,87t + \varphi) - 1,74\, \rho\, e^{-0,6t} \sin(0,87t+\varphi)$.

t=0 $\Rightarrow \dot{Y}(0) = -3,3A_1 - 1,2\rho\cos\varphi - 1,74\rho\sin\varphi$.

Derivam din nou si obtinem:

$\ddot{Y}(t) = 10,9A_1 e^{-3,3t} + 0,72\rho\, e^{-0,6t} \cos(0,87t + \varphi) - 1,044\, \rho\, e^{-0,6t} \sin(0,87t+\varphi) +$
$1,044\, \rho\, e^{-0,6t} \sin(0,87t+\varphi) - 1,51\, \rho\, e^{-0,6t} \cos(0,87t+\varphi). \Rightarrow$

$\ddot{Y}(t) = 10,9A_1 e^{-3,3t} + 0,72\rho\, e^{-0,6t} \cos(0,87 + \varphi) - 1,51\, \rho\, e^{-0,6t} \cos(0,87t+\varphi)$.

b4) Decizia privind cheltuielile guvernamentale $\hat{G}(t)$ combinand efectul proportional cu efectul diferential

$\hat{G}(t) = -\gamma_P Y(t) - \gamma_D \dot{Y}(t)$ = -0,2Y(t)- 0,1$\dot{Y}(t)$

Relatia de echilibru dintre cererea si oferta agregata este de forma:

$$\ddot{Y}(t) + [\alpha + ks\gamma_D]\dot{Y}(t) + \alpha k(s + \gamma_P)Y(t) = -\alpha ku \Rightarrow$$

$$\ddot{Y}(t) + 3{,}15\dot{Y}(t) + 7{,}5Y(t) - 15 = 0$$

Ecuatia caracteristica se scrie:

$\lambda^2 + 3{,}15\lambda + 7{,}5 = 0$

$\lambda_P = 0{,}2 > \frac{(\alpha - ks)^2}{4\alpha k} = \frac{1{,}5^2}{60} = 0{,}037$, deci $\Delta < 0$ si $\lambda_{1,2} \in C$, $\lambda_{1,2}$ =a$\pm bi$

$\Delta = -20$

$\lambda_{1,2} = \frac{-3{,}15 \pm 4{,}48i}{2}$;

Y(t) = $Y^G(t) + Y^*$ = $A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$ + Y^*,

In care

$e^{\lambda_1 t}$ = $e^{\frac{-3{,}15}{2}t} e^{\frac{4{,}48}{2}it} = e^{\frac{-3{,}15}{2}t}\left(\cos\frac{4{,}48}{2}t + i\sin\frac{4{,}48}{2}t\right)$

$e^{\lambda_2 t}$ = $e^{\frac{-3{,}15}{2}t}\left(\cos\frac{4{,}48}{2}t - i\sin\frac{4{,}48}{2}t\right)$

si $Y^* = -2{,}4$

Coeficientii $A_{1,} A_2$ se determina din datele initiale:

t = 0$\Rightarrow$ $A_{1,} + A_2$ +`Y^* =Y(0) (5)

Derivand, obtinem : $\dot{Y}(t) = A_1 \lambda_1 e^{\lambda_1 t} + A_2 \lambda_2 e^{\lambda_2 t}$ (6)

Din (5) si (6) obtinem sistemul

$$\begin{cases} A_1 + A_2 = Y(0) - Y^* \\ A_1\lambda_1 + A_2\lambda_2 = \dot{Y}(0) \end{cases}$$

prin inlocuire obtinem

$$\begin{cases} A_1 + A_2 = 2002,4 \\ (\frac{-3,15+4,48i}{2})A_1 + (\frac{-3,15-4,48i}{2})A_2 = -400 \end{cases}$$

Obtinem:

$$\begin{cases} A_1 = 800 - 535i \\ A_2 = 800 + 535i \end{cases}$$

Avem $\rho = |A_1| = \sqrt{800^2 + 535^2} = 962$

Y(t) = $Y^G(t) + Y^*$ = (800-535i)$e^{(\frac{-3.15+4,48i}{2})t}$ $+(800+535i)\ e^{(\frac{-3,15-4,48}{2})t}$ -2,4

Cum a = -3,15<0 $\Rightarrow$ Dinamica lui Y(t)este de tip stationar (oscilant amortizata), deci cand $\gamma_D = 0,1 < \dfrac{\alpha + ks}{ks} = 0,3$

Cum $\gamma_D \in \left(\dfrac{(\sqrt{\alpha} - \sqrt{ks})^2}{\alpha k}, \dfrac{\alpha + ks}{\alpha k} \right)$=(0,01;0,3) , dinamica lui Y(t) este oscilant amortizata in jurul traiectoriei de echilibru sationar Y^*.

Evolutia ofertei este oscilanta cu perioada T = $\dfrac{2\pi}{b} = \dfrac{2*3,14}{4,48} = 1,4$ ani cu amplitudinea exponential descrescatoare.

Se constata o reducere a ofertei cu -2,4 unitati indusa de reducerea cererii cu o unitate , fata de cazul initial , cand nu se asigura o politica guvernamentala de stabilizare, cand reducerea era de 5 uniati.

b5) Decizia privind cheltuielile guvernamentale $\hat{G}(t)$ combinand efectul proportional cu efectul integral

$$\hat{G}(t) = -\gamma_P Y(t) - \gamma_I \int_0^t Y(\tau)d\tau = \text{-0,2Y(t)- 0,25}\, \gamma_I \int_0^t Y(\tau)d\tau$$

Relatia de echilibru dintre cererea si oferta agregata este de forma:

$$\dddot{Y}(t) + [\alpha + ks\ ]\ddot{Y}(t) + \alpha k(s+\gamma_P)\dot{Y}(t) + \alpha k\gamma_I[Y(t) - Y_0] = 0 \ \Rightarrow$$

$$\dddot{Y}(t) + 4{,}5\ddot{Y}(t) + 7{,}5\dot{Y}(t) + 3{,}75Y(t) - 3{,}75Y(0) = 0$$

Ecuatia caracteristica asociata ecuatiei omogene obtinuta se scrie

$$\lambda^3 + [\alpha + ks]\lambda^2 + \alpha k(s+\gamma_P)\ \lambda + \alpha k\gamma_I\ = 0$$
$$\lambda^3 + 4{,}5\lambda^2 + 7{,}5\ \lambda + 3{,}75\ = 0$$

Pentru analiza stabilitatilor radacinilor ecuatiei se poat eutiliza criteriul matricial de stabilitate al lui Routh-Hurwitz.Conform acestuia, ecuatia algebrica de gradul trei

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3\ = 0$$

$$\lambda^3 + 4{,}5\lambda^2 + 7{,}5\ \lambda + 3{,}75\ = 0$$

are radacinile subunitare (positive sau negative), daca matricea de stabilitate

$$S = \begin{pmatrix} a_1 a_3 a_5 \\ a_0 a_2 a_4 \\ 0 a_1 a_3 \end{pmatrix}$$

are toti minorii principali pozitivi ($\Delta_i > 0$) .

Astfel:

$$S = \begin{pmatrix} 4{,}5 & 3{,}75 & 0 \\ 1 & 7{,}5 & 0 \\ 0 & 4{,}5 & 3{,}75 \end{pmatrix}$$

$\Delta_1 = \alpha + ks = 4{,}5 > 0$

$$\Delta_2 = \begin{vmatrix} 4{,}5 & 3{,}75 \\ 2 & 7{,}5 \end{vmatrix} > 0$$

$\Delta_3 > 0$

Punand conditia ca $\Delta_2 > 0$ avem relatia care da stabilitatea in acest caz:

$\gamma_I < (s + \gamma_P)(\alpha + ks) = 2{,}7$

Doarece s + γ_P >s, rezulta ca domeniul de stabilitate al parametrului γ_I, in conditiile acestei politici, este mai mare decat cel in cazul politicii integrale simple.

$\lambda_1 = -2{,}5 \Rightarrow \lambda^2 + 2\lambda + 2{,}5 = 0$, cu $\lambda_{2,3} \approx -1 \pm 1{,}22i$

Evolutia ofertei agregate este data de :

$$Y(t) = A_1 e^{-2{,}5t} + 2\rho\, e^{-t} \cos(1{,}22t + \varphi) + Y^*$$

unde $\rho = |c_2|$, $\varphi = \arg c_2$

t=0 $\Rightarrow A_1 + 2\rho \cos\varphi = Y_0 - Y^*$

Dupa ce derivam, obtinem:

$\dot{Y}(t) = -2{,}5A_1 e^{-2{,}5t} - 2\rho\, e^{-t} \cos(1{,}22t + \varphi) - 2{,}44\, \rho\, e^{-t} \sin(1{,}22t + \varphi)$.

t=0 $\Rightarrow \dot{Y}(0) = -2{,}5A_1 - 2\rho \cos\varphi - 2{,}44\rho \sin\varphi$.

Derivam din nou si obtinem:

$\ddot{Y}(t) = 6{,}25A_1 e^{-2{,}5t} + 2\rho\, e^{-t} \cos(1{,}22t + \varphi) - 2{,}44\, \rho\, e^{-t} \sin(1{,}22t + \varphi) +$
$2{,}44\, \rho\, e^{-t} \sin(1{,}22t + \varphi) - 2{,}97\, e^{-t} \cos(1{,}22t + \varphi). \Rightarrow$

$\ddot{Y}(t) = 6{,}25A_1 e^{-2{,}5t} + 2\rho\, e^{-t} \cos(1{,}22t + \varphi) - 2{,}97\, e^{-t} \cos(1{,}22t + \varphi)$.

b6) Decizia privind cheltuielile guvernamentale $\hat{G}(t)$ combinand efectul decizional cu efectul integral

$$\hat{G}(t) = -\gamma_D \dot{Y}(t) - \gamma_I \int_0^t Y(\tau)d\tau = -0{,}1Y(t) - 0{,}25\,\gamma_I \int_0^t Y(\tau)d\tau$$

Relatia de echilibru dintre cererea si oferta agregata este de forma:

$$\ddot{Y}(t) + (\alpha + ks + \gamma_D)\dot{Y}(t) + \alpha ks Y(t) + \alpha k \gamma_I \int_0^t Y(\tau)d\tau = -\alpha k u$$

$$\dddot{Y}(t) + [\alpha + ks \ + \gamma_D]\ddot{Y}(t) + \alpha ks \dot{Y}(t) + \alpha k \gamma_I [Y(t) - Y_0] = 0 \ \Rightarrow$$

$$\dddot{Y}(t) + 4{,}6\ddot{Y}(t) + 4{,}5\dot{Y}(t) + 3{,}75Y(t) - 3{,}75Y(0) = 0$$

Ecuatia caracteristica asociata ecuatiei omogene obtinuta se scrie

$$\lambda^3 + 4{,}6\lambda^2 + 4{,}5\ \lambda + 3{,}75\ = 0$$

$$S = \begin{pmatrix} 4{,}6 & 3{,}75 & 0 \\ 1 & 4{,}5 & 0 \\ 0 & 4{,}5 & 3{,}75 \end{pmatrix}$$

$$\Delta_1 = \alpha + ks = 4{,}6 > 0$$

$$\Delta_2 = \begin{vmatrix} 4{,}6 & 3{,}75 \\ 3 & 4{,}5 \end{vmatrix} > 0$$

$$\Delta_3 > 0$$

$\lambda_1 = -3{,}75 \Rightarrow \lambda^2 + 0{,}85\lambda + 1{,}32 = 0$, cu $\lambda_{2,3} \approx -0{,}42 \pm 1{,}06i$

Evolutia ofertei agregate este data de :

$$Y(t) = A_1 e^{-3,75t} + 2\rho\, e^{-0,42t} \cos(1{,}06t + \varphi) + Y^*$$

unde $\rho = |c_2|$, $\varphi = \arg c_2$

t=0 $\Rightarrow A_1 + 2\rho \cos\varphi = Y_0 - Y^*$

Dupa ce derivam, obtinem:

$\dot{Y}(t) = -3{,}75 A_1 e^{-3,75t} - 2\rho\, e^{-0,42t} \cos(1{,}06t + \varphi) - 2{,}12\,\rho\, e^{-0,42t} \sin(1{,}06t + \varphi)$.

t=0 $\Rightarrow \dot{Y}(0) = -3{,}75A_1 - 2\rho\cos\varphi - 2{,}12\rho\sin\varphi$.

Derivam din nou si obtinem:

$\ddot{Y}(t) = 14{,}06 A_1 e^{-3,75t} + 2\rho\, e^{-0,42t} \cos(1{,}06t + \varphi) - 2{,}12\,\rho\, e^{-0,42t} \sin(1{,}06t + \varphi) +$
$2{,}12\,\rho\, e^{-0,42t} \sin(1{,}06t + \varphi) - 2{,}24\, e^{-0,42t} \cos(1{,}06t + \varphi). \Rightarrow$

$\ddot{Y}(t) = 14{,}06 A_1 e^{-3,75t} + 2\rho\, e^{-t} \cos(1{,}06t + \varphi) - 2{,}24\, e^{-t} \cos(1{,}06t + \varphi)$.

Observam ca satbilitatea se obtine punand $\Delta_1 > 0$ si $\Delta_2 > 0$.Din prima conditie obtinem:

$$\gamma_D > -(\alpha + ks) = -4{,}6,$$

iar din a doua coditie obtinem:

$$\alpha ks(\alpha + ks + \gamma_D) - \alpha k \gamma_I > 0$$

$\gamma_I < \dfrac{\alpha ks(\alpha + ks + \gamma_D)}{\alpha k} \Rightarrow \gamma_I <$ s($\alpha + ks + \gamma_D$)=0,3(3+5*0,3+0,1)=1,38

Aceste doua conditii asupra parametrilor de comanda $\lambda_D si \lambda_I$ definesc domeniul de stabilitate al sistemului.